

A posteriori error estimates for elliptic problems with Dirac measure terms in weighted spaces

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Abstract

In this article we develop a posteriori error estimates for general second order elliptic problems with point sources in two- and three-dimensional domains. We prove a global upper bound and a local lower bound for the error measured in a weighted Sobolev space. The weight considered is a (positive) power of the distance to the support of the Dirac delta source term, and belongs to the Muckenhoupt's class A_2 . The theory hinges on local approximation properties of either Clément or Scott-Zhang interpolation operators, without need of suitable modifications, and makes use of weighted estimates for fractional integrals and maximal functions. Numerical experiments with an adaptive algorithm yield optimal meshes and very good effectivity indices.

Keywords: elliptic problems, point sources, a posteriori error estimates, finite elements, weighted Sobolev spaces

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