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CONTINUOUS TIME RANDOM WALKS AND THE CAUCHY PROBLEM FOR THE HEAT EQUATION

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ABSTRACT. In this paper we deal with anomalous diffusions induced by Continuous Time Random Walks - CTRW in $\mathbb{R}^n$. A particle moves in $\mathbb{R}^n$ in such a way that the probability density function $u(\cdot,t)$ of finding it in region Ω of $\mathbb{R}^n$ is given by $\int_\Omega u(x,t)dx$. The dynamics of the diffusion is provided by a space time probability density $J(x,t)$ compactly supported in $\{t \geq 0\}$. For t large enough, u must satisfy the equation $u(x,t) = [(J-\delta) * u](x,t)$ where δ is the Dirac delta in space time. We give a sense to a Cauchy type problem for a given initial density distribution f. We use Banach fixed point method to solve it, and we prove that under parabolic rescaling of J the equation tends weakly to the heat equation and that for particular kernels J the solutions tend to the corresponding temperatures when the scaling parameter approaches to zero.