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NONLOCAL DIFFUSIONS ON FRACTALS. QUALITATIVE PROPERTIES AND NUMERICAL APPROXIMATIONS.

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ABSTRACT. We propose a numerical method to approximate the solution of a nonlocal diffusion problem on a general setting of metric measure spaces. These spaces include, but are not limited to, fractals, manifolds and Euclidean domains. We obtain error estimates in $L^{\infty}(L^p)$ for $p = 1, \infty$ under the sole assumption of the initial datum being in L^p . An improved bound for the error in $L^{\infty}(L^1)$ is obtained when the initial datum is in L^2 . We also derive some qualitative properties of the solutions like stability, comparison principles and study the asymptotic behavior as $t \to \infty$. We finally present two examples on fractals: the Sierpinski gasket and the Sierpinski carpet, which illustrate on the effect of nonlocal diffusion for piecewise constant initial datum.