

BOUNDEDNESS AND COMPACTNESS FOR COMMUTATORS
OF SINGULAR INTEGRALS RELATED TO A CRITICAL
RADIUS FUNCTION

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ABSTRACT. We work in the general framework of a family of singular integrals with kernels controlled in terms of a critical radius function ρ . This family models the harmonic analysis derived from the Schrödinger operator $L = -\Delta + V$, where the non-negative potential V satisfies an appropriate reverse Hölder condition. For their commutators, we find sufficient conditions on the symbols for boundedness and/or compactness when acting on weighted L^p spaces. In all cases, the classes of symbols and weights are larger than their classical counterparts, BMO, CMO and A_p . When these general results are applied to the Schrödinger context, we obtain boundedness and compactness for commutators of operators like $\nabla L^{-1/2}$, $\nabla^2 L^{-1}$, $V^{1/2} L^{-1/2}$, $V^{1/2} \nabla L^{-1}$, $V L^{-1}$ and L^{loc} . As in Uchiyama's classical paper, we give a full description of the class for compactness, CMO_ρ^∞ , assuming ρ to be bounded. Finally, we provide examples showing that CMO is strictly contained in CMO_ρ^∞ for any ρ , bounded or not.