Abstract

The third order hypermetric $\rho(x,y,z)$ in $\mathbb{R}^n$ is given by the Euclidean distance of the point $(x,y,z) \in (\mathbb{R}^n)^3$ to the diagonal $\Delta^{(3)} = \{(x,x,x) : x \in \mathbb{R}^n\}$ of $(\mathbb{R}^n)^3$. In this paper, we consider local and nonlocal fractional versions of bilinear differential operators generated by the geometry induced by $\rho(x,y,z) = \rho(0,x-y,x-z)$ instead of the usual $|(x-y,x-z)|$, where $|\cdot|$ denotes the Euclidean norm in $\mathbb{R}^{2n}$. In doing so, we obtain a fractional nonlocal version $\mathbb{D}^s(f,g)$, $0 < s < 1$, of the Dirichlet bilinear operator $\nabla f(x) \cdot \nabla g(x)$ which converges to this classical form when the fractional parameter approaches its endpoint $s = 1$. We consider both the kernel approach and the Fourier multiplier version.